

Solution

Physics 214

Problem 2

Week 7

Is the Electron Ever Inside the Proton?

The proton has a radius of $r_p = 0.7 \times 10^{-15} \text{ m}$ ($\equiv 0.7 \text{ femtometers}^*$, or 0.7 fm). The electron in the ground state of the hydrogen atom is described by the wave function,

$$\psi_{1S}(r) = \frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o} \quad a_o = 0.053 \text{ nm is the Bohr radius.}$$

- a) What is the probability of finding the electron inside the proton?

Hint: In three dimensions ψ^2 has units of probability per meter³. Also, $r_p \ll a_o$.

Because $r_p \ll a_o$, ψ is nearly constant: $\Psi^2(0) = 1/\pi a_o^3$. So, the probability is:

$$P(\text{inside}) = \frac{1}{\pi a_o^3} \frac{4\pi r_p^3}{3} = \frac{4}{3} \left(\frac{r_p}{a_o} \right)^3 = 3.07 \times 10^{-15}.$$

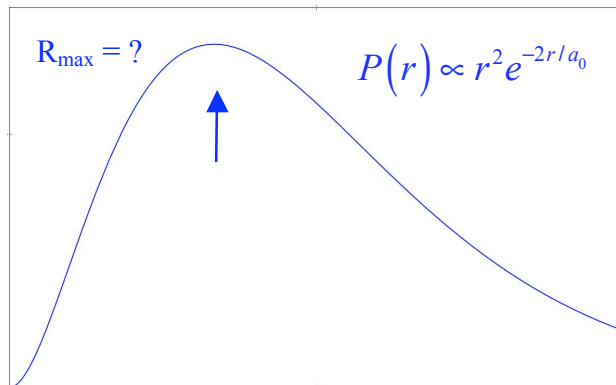
The electron is occasionally inside the proton, but the probability is tiny.

COMMENT: For heavy atoms (e.g., gold) the probability is much larger, because the orbit is smaller, and the nucleus is bigger.

- b) Near what distance from the nucleus, r , are you most likely to find the electron? Hint: The probability of finding the particle within a thin shell of radius r and thickness dr is

$|\psi(r)|^2 d\text{Vol} = |\psi(r)|^2 (4\pi r^2 dr) \equiv P(r) dr$. Sketch $P(r)$. How do you find the maximum of this?

This probability will be maximum at the peak. To find this, we simply set $dP(r)/dr = 0$:



$$\frac{dP(r)}{dr} \propto \frac{d}{dr} (r^2 e^{-2r/a_o}) = (2r) e^{-2r/a_o} + r^2 \left(\frac{-2}{a_o} e^{-2r/a_o} \right) = r e^{-2r/a_o} \left(1 - \frac{r}{a_o} \right) = 0.$$
$$\therefore r = a_o$$

* The femtometer [fm] is also called the “fermi”, after Enrico Fermi, the eponym for Fermilab.